# Temperature effects in ultrasonic Lamb wave structural health monitoring systems

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(Received 19 February 2008; revised 24 April 2008; accepted 27 April 2008)

There is a need to better understand the effect of temperature changes on the response of ultrasonic guided-wave pitch-catch systems used for structural health monitoring. A model is proposed to account for all relevant temperature-dependent parameters of a pitch-catch system on an isotropic plate, including the actuator-plate and plate-sensor interactions through shear-lag behavior, the piezoelectric and dielectric permittivity properties of the transducers, and the Lamb wave dispersion properties of the substrate plate. The model is used to predict the  $S_0$  and  $A_0$  response spectra in aluminum plates for the temperature range of -40-+60 °C, which accounts for normal aircraft operations. The transducers examined are monolithic PZT-5A [PZT denotes Pb(Zr-Ti)O<sub>3</sub>] patches and flexible macrofiber composite type P1 patches. The study shows substantial changes in Lamb wave amplitude response caused solely by temperature excursions. It is also shown that, for the transducers considered, the response amplitude changes follow two opposite trends below and above ambient temperature effects in guided-wave damage detection systems.

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PACS number(s): 43.35.Zc, 43.35.Yb, 43.40.Le, 43.38.Fx [YHB]

**I. INTRODUCTION** 

With the advent of condition-based maintenance of structures, structural health monitoring (SHM) systems using attached permanently piezoelectric patches often  $Pb(Zr-Ti)O_3$  (PZT)] for ultrasonic guided-wave testing are becoming increasingly popular. While the wave actuation and sensing properties of PZT elements have been studied extensively, the effect of temperature variation is much less understood. However, temperature effects are critical in structures such as aircraft which can experience changes from  $-40 \,^{\circ}\text{C}$  (during high altitude flights) to  $+60 \,^{\circ}\text{C}$  (during storage in closed hangers). What is missing is a model that includes the cumulative role of transducer elements (actuator and sensor), substrate structure, and transducer/structure interaction to predict the full pitch-catch guided-wave response under changing temperature.

One of the earliest works on the temperature effects on Lamb wave signals was performed by Blaise and Chang (2001) who considered a pitch-catch arrangement between 50 and 150 kHz on sandwich panels at low temperatures (-90 °C). The authors observed a decrease in wave amplitude and time of flight from ambient temperature to -90 °C, and they developed an empirical model to fit the experimental data. Lee *et al.* (2003) examined experimentally the effect of high temperature values (35–70 °C) on the response of PZT patches in an  $S_0$  pitch-catch configuration on an aluminum plate. They measured a decrease in  $S_0$  amplitude with increasing temperature, and they suggested different feature extraction strategies to detect damage in a changing temperaPages: 161-174

Additional work on temperature effects on guided-wave SHM systems was performed in the area of reverberating signals. Konstantinidis et al. (2006) isolated the variation in wave amplitude between 22 and 32 °C for various portions of Lamb wave measurements in aluminum plates, specifically coherent noise, first arrivals of  $S_0$  and  $A_0$  modes, and their edge reflections. They experimentally observed that temperature changes result in shifts in wave arrival times and in center frequency of the received waves under constant excitation frequency. These effects were explained by shifts in the wave dispersion curves due to thermal expansion and elastic properties of the plate. Lu and Michaels (2005) and Michaels and Michaels (2005) examined the difference in various damage-sensitive features of diffuse Lamb wave signals at temperatures varying between 5 and 40 °C. Their main experimental observation was a changing time of flight, which was attributed to thermal expansion and Young's modulus of the substrate. They recommended adopting a

ture environment. Chambers *et al.* (2006) acquired pulseecho measurements in aluminum plates by using Lamb wave transducers by Metis Design Corporation at high temperature (85 °C). A drop in wave amplitude from room temperature was observed, and it was justified by a degradation of the couplant between the plate and the clamp used to produce a wave reflection. Extremely high temperatures, up to 230 °C, were considered by Schulz *et al.* (2003) who used PZT-5A patches as free vibration sensors bonded on aluminum beams by using various adhesives. It was experimentally observed that the sensing performance of the patches degraded with increasing temperature. This was attributed to the loss of piezoelectric properties through depoling and to the increased compliance of the patch-beam adhesive layer.

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FIG. 1. Problem statement: pitch-catch Lamb wave monitoring by piezo-electric patches.

"bank" of baseline signals acquired at the various temperatures to discriminate the effects of damage from those of environmental changes.

Among the most comprehensive studies on temperature effects on guided-wave signals from PZT patches is the work by Raghavan and Cesnik (2007). This paper examined the pitch-catch response of PZT-5A patches for temperature range internal to spacecraft structures (20-150 °C), limited to the  $S_0$  mode at the specific frequency of 120 kHz. Pitchcatch measurements were taken on an aluminum plate in an autoclave showing an increase in  $S_0$  time of flight with increasing temperature and a nonmonotonic amplitude change (increasing amplitude from 20 to 90 °C and decreasing amplitude from 90 to 150 °C). A theoretical model was developed to corroborate the experiment. The temperature dependence of several parameters was included in the model. The PZT/structure interaction was modeled by perfect-bond conditions, i.e., the classical pin-force model with two Dirac functions for the shear stress at the PZT actuator ends (Raghavan and Cesnik 2005). The properties of the bond layer were considered to be constant with temperature, and shear-lag effects were approximated by the reduction in PZT length ("effective length"). While the agreement between model and experiment was found to be satisfactory for the wave time-of-flight results, the discrepancies found for the wave amplitude results were attributed by the authors to the assumption of perfect bond and constant adhesive layer properties through the temperature range.

The present study continues the work on the temperature effects in Lamb-wave SHM systems. The full temperature range typical of aircraft operations  $(-40 - +60 \degree C)$  is considered in a large Lamb frequency range of  $\sim 100-500$  kHz. The proposed temperature-dependent model combines the steps of (a) wave generation by the actuator patch, (b) wave propagation in the substrate plate, and (c) wave detection by the sensor patch. Actuator-plate and sensor-plate interactions are modeled by shear-lag theories. The study considers monolithic PZT patches and macrofiber composite (MFC) patches. MFC patches, originally developed at NASA Langley Research Center for low-frequency structural control (Wilkie et al. 2000, Sodano et al. 2004), are recently being used for guided-wave transduction owing to their superior flexibility and durability compared to monolithic PZTs (Thien et al. 2005, Lanza di Scalea et al. 2007b, Matt and Lanza di Scalea 2007).

# **II. PROBLEM STATEMENT**

The problem examined (Fig. 1) is that of a piezoelectric patch actuator on an isotropic (aluminum) plate exciting the fundamental Lamb modes which are detected by a similar



FIG. 2. Schematics for (a) monolithic PZT patch, (b) composite MFC type P1 patch, and (c) equivalent model for MFC type P1 sensor patch.

piezoelectric patch sensor (pitch catch). The cases of monolithic PZT patches and MFC patches are considered. The problem is studied in two dimensions and assuming plane waves.

#### **III. ACTUATION STRAIN**

#### A. Monolithic PZT actuator

The strain generated by an excitation voltage in a monolithic PZT actuator is governed by the converse piezoelectric effect (IEEE 1978),

$$\boldsymbol{\varepsilon} = -\boldsymbol{d}\boldsymbol{E} + \boldsymbol{S}\boldsymbol{\sigma},\tag{1}$$

where  $\varepsilon$  is the strain vector (6×1), d is the piezoelectric coefficient matrix (6×3), E is the electric field vector (3×1), S is the elastic compliance matrix (6×6), and  $\sigma$  is the stress vector (6×1).

Assuming in-plane axes (1, 2) and out-of-plane axis 3 for the PZT actuator [Fig. 2(a)], the actuated normal strains from Eq. (1) are given by

$$\begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \end{bmatrix} = \begin{bmatrix} 0 & 0 & d_{31} \\ 0 & 0 & d_{32} \\ 0 & 0 & d_{33} \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix}.$$
 (2)

For monolithic PZTs  $(d_{31}=d_{32})$ , poled along the thickness direction 3  $(E_1=E_2=0)$ , the actuation strain  $\varepsilon_{PZT}$  simplifies to

$$\varepsilon_{\text{PZT}} = \varepsilon_{11} = d_{31}E_3 = -d_{31}\frac{V_{\text{appl}}}{t_{\text{PZT}}},\tag{3}$$

where  $V_{appl}$  is the applied voltage and  $t_{PZT}$  is the thickness of the actuator.

#### **B.** Composite MFC actuator

MFC type P1 transducers are composed of PZT fibers that are unidirectionally aligned, embedded into an epoxy matrix, and sandwiched between two sets of interdigitated electrode patterns (Smart Material Corporation, Saratoga, FL). Because of their polymer-based composite design, MFC transducers are more flexible and durable than monolithic PZT transducers. The model in Fig. 2(b) shows the in-plane axes (1, 2), out-of-plane axis 3, interelectrode spacing a, and electrode length  $l_e$ . The actuated normal strain for the MFC is given by

$$\begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \end{bmatrix} = \begin{bmatrix} d_{11} & 0 & 0 \\ d_{12} & 0 & 0 \\ d_{13} & 0 & 0 \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix}.$$
 (4)

Poling is now along the in-plane (fiber) direction 1 ( $E_2 = E_3 = 0$ ), and the actuation strain  $\varepsilon_{\text{MFC}}$  is given by



FIG. 3. (a) Shear-lag actuator model and (b) shear-lag sensor model.

$$\varepsilon_{\rm MFC} = \varepsilon_{11} = d_{11}E_1 = -d_{11}\frac{V_{\rm appl}}{a}.$$
(5)

The equivalence between a monolithic PZT actuator and a type P1 MFC actuator is, thus, readily obtained by considering the appropriate piezoelectric coefficient ( $d_{11}$  versus  $d_{31}$ ) and replacing the PZT thickness  $t_{PZT}$  with the MFC interelectrode spacing a.

# **IV. ACTUATED STRAIN IN THE PLATE**

#### A. Actuated shear stress

The strain generated in the plate by the actuator is here modeled by coupling the shear-lag theory of Crawley and de Luis (1987) with the Lamb wave generation model by Giurgiutiu (2005). The present model is, therefore, complementary to these previous studies because the former work was not concerned with wave generation or detection, and the latter work did not consider shear-lag phenomena.

Assuming the surface-bonded actuator system in Fig. 3(a), the nondimensional shear-lag parameter  $\Gamma$  from Crawley and de Luis (1987) is given by

$$\Gamma^{2} = \frac{G_{\text{bond}}}{Y_{\text{act}}^{E}} \frac{l_{\text{act}}^{2}}{t_{\text{bond}} t_{\text{act}}} \frac{\Psi + \alpha}{\Psi},$$
(6)

where  $G_{\text{bond}}$  and  $t_{\text{bond}}$  are the shear modulus and thickness of the adhesive layer,  $Y_{\text{act}}^E$ ,  $l_{\text{act}}$ , and  $t_{\text{act}}$  are the in-plane Young's modulus, length, and thickness of the PZT or MFC actuator, the factor  $\alpha$  is 1 or 3 for extensional or bending actuation, respectively, and  $\Psi$  is the plate-actuator stiffness ratio given by

$$\Psi = \frac{E_{\text{plate}}t_{\text{plate}}}{Y_{\text{act}}^E t_{\text{act}}}.$$
(7)

It should be said that the above values of  $\alpha$  rely on low-frequency plate theory and are not necessarily applicable to high frequency times plate thickness products. Substitution of Eq. (7) into Eq. (6) yields

$$\Gamma^{2} = \frac{G_{\text{bond}} l_{\text{act}}^{2}}{t_{\text{bond}}} \left( \frac{1}{Y_{\text{act}}^{E} t_{\text{act}}} + \frac{\alpha}{E_{\text{plate}} t_{\text{plate}}} \right).$$
(8)

It is well-known that the shear-lag effect increases with increasing compliance and thickness of the bond layer. Perfect bonding (no shear lag) is achieved for a thin and stiff bond layer where  $\Gamma \rightarrow \infty$ .



FIG. 4. In-plane shear stress at the actuator/plate interface under shear-lag conditions: (a) changing bond layer stiffness and (b) changing actuator stiffness. Insets zoom near the end of the actuator.

In a general shear-lag condition, Crawley and de Louis' (1987) distribution of in-plane shear stresses applied by the actuator to the bond layer into the plate,  $\tau(x)$ , can be rewritten as

$$\tau(x) = -\frac{G_{\text{bond}} l_{\text{act}} \varepsilon_{\text{act}}}{t_{\text{bond}} \Gamma \cosh \Gamma} \sinh \left[ \Gamma\left(\frac{2x}{l_{\text{act}}}\right) \right] = A \sinh(Bx), \quad (9)$$

where the actuation strain is  $\varepsilon_{act} = \varepsilon_{PZT}$  in Eq. (3) for a monolithic PZT and  $\varepsilon_{act} = \varepsilon_{MFC}$  in Eq. (5) for a MFC. The variables *A* and *B* are

$$A = -\frac{G_{\text{bond}} l_{\text{act}} \varepsilon_{\text{act}}}{t_{\text{bond}} \Gamma \cosh \Gamma}, \quad B = \frac{2\Gamma}{l_{\text{act}}}.$$
 (10)

Figure 4(a) shows the effect of changing bond layer stiffness on the generated shear stress. Figure 4(b) shows, instead, the effect of changing plate/actuator stiffness ratio. Both cases consider a baseline PZT-5A actuator  $(Y_{act}^{E}=70 \text{ GPa}, l_{act}=10 \text{ mm}, t_{act}=0.63 \text{ mm}, d_{31}=-168 \times 10^{-12} \text{ m/V})$ , excited by a unity voltage  $(V_{appl}=1 \text{ V})$ , bonded by an adhesive layer  $(t_{bond}=0.01 \text{ mm})$  on an aluminum plate  $(E_{plate}=70 \text{ GPa}, G_{plate}=27 \text{ GPa}, t_{plate}=1.58 \text{ mm})$ . Extensional actuation is considered  $(\alpha=1)$ .

In Fig. 4(a),  $\tau(x)$  from Eq. (9) is plotted for a nominal bond layer stiffness of  $G_{bond}=2.8$  GPa (corresponding to a shear-lag parameter  $\Gamma=68.4$ ) and for two extreme cases of a very stiff adhesive ( $G_{bond}=28$  GPa or  $\Gamma=216.4$ ) and a very compliant adhesive ( $G_{bond}=0.28$  GPa or  $\Gamma=21.6$ ). The PZT actuator stiffness is maintained constant at  $Y_{act}^{E}=70$  GPa. This figure shows the known result that a stiffer adhesive increasingly confines the shear stress transfer toward the ac-

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tuator's ends. However, the different adhesive stiffnesses maintain a roughly constant area under the  $\tau(x)$  curve [see inset of Fig. 4(a)], indicating little change in the net force transferred to the plate.

In Fig. 4(b),  $\tau(x)$  is plotted for a constant  $G_{\text{bond}} = 2.8$  GPa and with changing actuator stiffness from a nominal  $Y_{\text{act}}^E = 70$  GPa  $(E_{\text{plate}}/Y_{\text{act}}^E = 1)$  to a very stiff actuator  $(Y_{\text{act}}^E = 210 \text{ GPa or } E_{\text{plate}}/Y_{\text{act}}^E = 0.33)$  and a very compliant actuator  $(Y_{\text{act}}^E = 7 \text{ GPa or } E_{\text{plate}}/Y_{\text{act}}^E = 10)$ . The most relevant effect in this case is the change in the area under the  $\tau(x)$  curve [see inset of Fig. 4(b)], indicating a smaller force transfer with decreasing stiffness of the actuator.

The shear stress distribution of Eq. (9) provides the excitation of the Lamb waves generated in the plate. This problem was studied by Giurgiutiu (2005) by taking a spatial Fourier transform on the excitation to move to the wave number domain k. The perfect-bond condition assumed by Giurgiutiu (2005), i.e., two Dirac functions for  $\tau(x)$  at the actuator's ends, readily admits a closed-form spatial Fourier transform. It is here shown that a closed-form Fourier transform  $\tilde{\tau}(k)$  can also be obtained in the more general shear-lag case, where  $\tau(x)$  is a hyperbolic sine function [Eq. (9)], once the finite length of the actuator is taken into account. The Fourier transform linearity property  $(Zf(x)|_k = Zf(x)|_k)$  and scaling property  $(f(Zx)|_k = 1/|Z|f(x)|_{k/Z})$  are first invoked. Considering that the shear stress  $\tau(x)$  is bounded by the actuator length in  $-l_{act}/2 \le x \le l_{act}/2$ , applying the scaling property also requires changing the integration limits of the Fourier transform to the new variable  $-Zl_{act}/2 \leq Zx$  $\leq Zl_{\rm act}/2$ ,

$$\widetilde{T}(k) = A \widetilde{\sinh(Bx)}|_{k} = \frac{A}{|B|} \int_{-Bl_{act}/2}^{Bl_{act}/2} \left(\frac{e^{x} - e^{-x}}{2}\right) e^{-i(k/B)x} dx$$

$$= \frac{A}{|B|} \left[\frac{e^{(Bl_{act}/2)(1-ik/B)} - e^{(-Bl_{act}/2)(1-ik/B)}}{2\left(1 - \frac{ik}{B}\right)} + \frac{e^{(-Bl_{act}/2)(1+ik/B)} - e^{(Bl_{act}/2)(1+ik/B)}}{2\left(1 + \frac{ik}{B}\right)}\right].$$
(11)

By substituting *A* and *B* from Eq. (10), using the properties of hyperbolic functions  $[\cosh(ix)=\cos x; \sinh(ix)=i\sin x]$ , Eq. (11) can be simplified to

$$\widetilde{\tau}(k) = -2i \frac{G_{\text{bond}} l_{\text{act}}^2}{t_{\text{bond}} (4\Gamma^2 + k^2 l_{\text{act}}^2)} \varepsilon_{\text{act}} \times \left[ k l_{\text{act}} \frac{\tanh(\Gamma)}{\Gamma} \cos\left(\frac{k l_{\text{act}}}{2}\right) - 2\sin\left(\frac{k l_{\text{act}}}{2}\right) \right].$$
(12)

Points of zero actuation are found by setting Eq. (12) to 0, which yields



FIG. 5. Magnitude of spatial Fourier transform of in-plane shear stress at the actuator/plate interface under shear-lag conditions: (a) changing bond layer stiffness and (b) changing actuator stiffness.

$$|\tilde{\tau}(k)| = 0$$
 (zero actuation) when  $\frac{\tan\left(k\frac{l_{act}}{2}\right)}{k\frac{l_{act}}{2}} = \frac{\tanh(\Gamma)}{\Gamma}.$ 
(13)

This equation can be solved numerically. For the perfectbond case  $(\Gamma \rightarrow \infty)$ , since  $\lim_{\Gamma \rightarrow \infty} \tanh(\Gamma)/\Gamma=0$ , points of zero actuation are found when  $\tan(kl_{act}/2)=0$  or  $kl_{act}/2$  $=n\pi$  for n=0,1,2... (i.e., actuator length equals an integer multiple of the wavelength  $\lambda$ ). This coincides with the known "wavelength tuning" result by Giurgiutiu (2005) under perfect-bond assumptions. Points of maximum actuation are, instead, found by setting  $\partial |\tilde{\tau}(k)| / \partial k = 0$  in Eq. (12). This equation can also be solved numerically. For perfect bond  $(\Gamma \rightarrow \infty)$ , one can demonstrate that the maximum actuation occurs for  $kl_{act}/2 = \pi(2n-1)/2$  (i.e., actuator length equals an odd multiple of the half wavelength  $\lambda/2$ ), which also retrieves Giurgiutiu's (2005) perfect-bond model.

The shear stress Fourier transform  $|\tilde{\tau}(k)|$  from Eq. (12) is plotted versus the nondimensional wave number  $(kl_{act}/2)$ , for the same cases previously presented in Fig. 4. Changing bond layer stiffness is shown in Fig. 5(a), and changing plate-actuator stiffness is shown in Fig. 5(b). In both graphs, the wavelength tuning conditions are evident. From Fig. 5(a), the predominant effect of a changing adhesive stiffness is a slight shift in zeroes and maxima of the  $|\tilde{\tau}(k)|$  curve, with little effect on the magnitude of  $|\tilde{\tau}(k)|$ . As discussed in the previous Fig. 4(a), a stiffer adhesive layer approaches perfect bond where the ideal wavelength tuning conditions above apply. A decreasing  $G_{\text{bond}}$  moves the shear stress transfer away from the actuator's ends. This decreases the "effective length" of the actuator  $l_{\text{eff}}$ , which shifts the  $|\tilde{\tau}(k)|$  curve toward increasing  $l_{\text{act}}$  (hence,  $k \times l_{\text{act}}$ ) to compensate for the shorter  $l_{\text{eff}}$ , hence, the shifts in Fig. 5(a). However, these shifts are not expected to be substantial for realistic ranges of adhesive properties. It is the actuator stiffness  $Y_{\text{act}}^E$  that influences highly the magnitude of the stress transfer, as shown in Fig. 5(b). Clearly,  $|\tilde{\tau}(k)|$  substantially decreases with degrading actuator stiffness.

Under temperature variations, both  $G_{\text{bond}}$  and  $Y_{\text{act}}^E$  will vary. As will be shown later, the temperature range of interest in this study (-40-+60 °C) will have a substantial influence on the actuation magnitude, with little change in the wavelength tuning points.

#### **B.** Generated Lamb strain

The Lamb strain generated by the actuator is found from the mode expansion formulation by Giurgiutiu (2005) coupled with the Fourier transform of the shear stress excitation in the shear-lag case derived in Eq. (12). Damping in the plate is neglected.

The time-domain in-plane strain at the surface of the plate and in the direction of wave propagation x is given using the residue theorem by

$$\varepsilon_{x}(x,t)|_{y=t_{\text{plate}}/2} = \frac{1}{2G_{\text{plate}}} \left[ \sum_{k^{S}} \tilde{\tau}(k^{S}) \frac{N_{S}(k^{S})}{\partial D_{S}(k^{S})/\partial k} e^{i(k^{S}x-\omega t)} + \sum_{k^{A}} \tilde{\tau}(k^{A}) \frac{N_{A}(k^{A})}{\partial D_{A}(k^{A})/\partial k} e^{i(k^{A}x-\omega t)} \right],$$
Antisymmetric modes
$$(14)$$

where  $k_i^S(\omega)$  are the eigenvalues for the Lamb symmetric modes (solutions of  $D_S=0$ ),  $k_i^A(\omega)$  are the eigenvalues for the Lamb antisymmetric modes (solutions of  $D_A=0$ ), and the remaining terms are the following well-known Rayleigh– Lamb expressions (Rose 1999):

$$N_{S}(k) = kq(k^{2} + q^{2})\cos\left(p\frac{t_{\text{plate}}}{2}\right)\cos\left(q\frac{t_{\text{plate}}}{2}\right),$$

$$D_{S}(k) = (k^{2} - q^{2})^{2}\cos\left(p\frac{t_{\text{plate}}}{2}\right)\sin\left(q\frac{t_{\text{plate}}}{2}\right)$$

$$+ 4k^{2}pq\sin\left(p\frac{t_{\text{plate}}}{2}\right)\cos\left(q\frac{t_{\text{plate}}}{2}\right),$$

$$N_{A}(k) = kq(k^{2} + q^{2})\sin\left(p\frac{t_{\text{plate}}}{2}\right)\sin\left(q\frac{t_{\text{plate}}}{2}\right),$$

$$D_{A}(k) = (k^{2} - q^{2})^{2}\sin\left(p\frac{t_{\text{plate}}}{2}\right)\cos\left(q\frac{t_{\text{plate}}}{2}\right)$$

$$+ 4k^{2}pq\cos\left(p\frac{t_{\text{plate}}}{2}\right)\sin\left(q\frac{t_{\text{plate}}}{2}\right),$$
(15)

$$p^2 = \left(\frac{\omega^2}{c_L^2} - k^2\right), \quad q^2 = \left(\frac{\omega^2}{c_T^2} - k^2\right),$$

with  $c_L$  and  $c_T$  as the bulk longitudinal and shear velocities, respectively.

By substituting  $\tilde{\tau}(k)$  from Eq. (12) into Eq. (14) and rearranging, the actuated surface strain in the plate can be written as

$$\varepsilon_x(x,t)\big|_{y=t_{\text{plate}}/2} = i\varepsilon_x e^{i(kx-\omega t)},\tag{16}$$

where the actuated wave amplitude is

$$\varepsilon_{x} = \frac{\tau_{1}}{2G_{\text{plate}}} \left[ \sum_{k^{S}} \tau_{2}(k^{S}) \frac{N_{S}(k^{S})}{\partial D_{S}(k^{S})/\partial k} \right] + \sum_{k^{A}} \tau_{2}(k^{A}) \frac{N_{A}(k^{A})}{\partial D_{A}(k^{A})/\partial k} \right].$$
(17)

In Eq. (17), the shear stress expression is decomposed into the following constant term,  $\tau_1$ , and k-dependent term,  $\tau_2(k)$ :

$$\tau_{1} = \frac{-2G_{\text{bond}}l_{\text{act}}^{2}\varepsilon_{\text{act}}}{t_{\text{bond}}},$$

$$\tau_{2}(k) = \frac{kl_{\text{act}}\frac{\tanh(\Gamma)}{\Gamma}\cos\left(\frac{kl_{\text{act}}}{2}\right) - 2\sin\left(\frac{kl_{\text{act}}}{2}\right)}{4\Gamma^{2} + k^{2}l_{\text{act}}^{2}}.$$
(18)

Figure 6 shows the actuated strain amplitude in the plate from Eq. (17) for the fundamental Lamb modes  $S_0$  and  $A_0$  as a function of the nondimensional wave number  $(kl_{act}/2)$ . The case of a monolithic PZT actuator in Fig. 6(a) is compared to that of a MFC type P1 actuator in Fig. 6(b). The graphs assume the properties of actuators, adhesive layer, and plate listed in Table I at ambient temperature. The excitation voltage is unity  $(V_{appl}=1 \text{ V})$ . In both graphs, the wavelength tuning conditions are evident. The amplitude of the actuated wave changes depending on the particular distribution of surface strain for  $S_0$  and  $A_0$ . The PZT actuator [Fig. 6(a)] generates  $S_0$  more efficiently than  $A_0$  at low kl values. The MFC actuator [Fig. 6(b)] generates  $A_0$  more efficiently than  $S_0$ throughout the kl range. Also noticeable is that the wave amplitude generated by the MFC actuator is one order of magnitude larger than that generated by the PZT actuator of the same length and for the same (unity) excitation voltage. This is primarily due to the actuator's geometrical properties (MFC interelectrode spacing a smaller than PZT thickness  $t_{act}$ ) and piezoelectric properties (MFC constant  $d_{11}$  larger than PZT constant  $d_{31}$ ), which both influence the actuation strains in Eqs. (3) and (5). Interestingly, it will be shown later that the geometrical parameters of interelectrode spacing for the MFC and actuator thickness for the PZT, which are relevant to the actuator generation efficiency, do not influence the actuator-sensor pitch-catch response.

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FIG. 6. Actuated strain amplitude of the fundamental  $S_0$  and  $A_0$  modes in a 1.58-mm-thick aluminum plate: (a) monolithic PZT actuator and (b) composite MFC-P1 actuator.

#### V. SENSOR RESPONSE

#### A. Shear-lag model for sensor

The shear-lag behavior considered for the actuator-plate interaction must now be extended to the plate-sensor interaction Fig. 3(b). To model the plate-*sensor* interaction, the equilibrium and compatibility conditions considered by Crawley and de Luis (1987) for their study of the *actuator*plate system can be applied. However, different boundary conditions must be considered in the case of the sensor, which lead to a different expression for the strain transferred to the sensor.

The shear-lag model by Crawley and de Luis (1987) leads to a pair of uncoupled fourth-order differential equa-

tions. As general solutions of these equations, the strain at the surface of the plate  $\varepsilon_{plate}$ , and the strain transferred to the sensor  $\varepsilon_{sens}$  are

$$\varepsilon_{\text{plate}} = B_1 + B_2 \frac{2x}{l_{\text{sens}}} + B_3 \sinh\left(\Gamma \frac{2x}{l_{\text{sens}}}\right) + B_4 \cosh\left(\Gamma \frac{2x}{l_{\text{sens}}}\right),$$
(19)
$$\varepsilon_{\text{sens}} = B_1 + B_2 \frac{2x}{l_{\text{sens}}} + \frac{\Psi}{\alpha} B_3 \sinh\left(\Gamma \frac{2x}{l_{\text{sens}}}\right)$$

$$- \frac{\Psi}{\alpha} B_4 \cosh\left(\Gamma \frac{2x}{l_{\text{sens}}}\right),$$

where  $l_{\text{sens}}$  is the sensor length in the wave propagation direction, and  $\Psi$  and  $\Gamma$  are the shear-lag terms given in Eqs. (7) and (8) with the substitution of the actuator properties with the sensor properties. The four integration constants  $B_i$ are found by applying the following boundary conditions at the ends of the sensor:

$$\varepsilon_{\text{plate}} = \varepsilon_x(\text{Lamb strain}) \text{ and } \varepsilon_{\text{sens}} = 0 \text{ at } x = \pm l_{\text{sens}}/2.$$
(20)

These assume that the strain in the plate is due to the Lamb waves and the sensor ends are free from normal stresses. These yield the following constants:

$$B_1 = \varepsilon_x \Psi / (\alpha + \Psi), \quad B_2 = B_3 = 0,$$

$$B_4 = \varepsilon_x \alpha \operatorname{sech}(\Gamma) / (\alpha + \Psi).$$
(21)

Notice that the integration constants for the sensor case are different from those derived by Crawley and de Luis (1987)for the actuator case. Substitution of Eq. (21) into Eq. (19) provides the following solution:

$$\varepsilon_{\text{sens}} = \varepsilon_x \left(\frac{\Psi}{\alpha + \Psi}\right) \left[1 - \frac{\cosh\left(\Gamma \frac{2x}{l_{\text{sens}}}\right)}{\cosh\Gamma}\right], \quad (22)$$

which is the final expression of the strain transferred to the sensor  $\varepsilon_{sens}$  as a function of the incoming Lamb strain in the plate  $\varepsilon_x$  and the shear-lag parameters  $\Psi$ ,  $\Gamma$ , and  $\alpha$ .

TABLE I. Ambient temperature properties of aluminum plate, adhesive layer, monolithic PZT patch, and composite MFC patch.

$E_{\text{plate}}$ =70 GPa		Aluminum plate $G_{\text{plate}}$ =27 GPa		$t_{\rm plate}$ =1.58 mm	
		Transducer-pla	te adhesive layer		
G <sub>bond</sub> =2.8 GPa			$t_{\rm bond}$ =0.01 mm		
$Y^E$ =70 GPa	ν=0.3	Monolithi $d_{31} = -168 \times 10^{-12}$ m/V	c PZT patch $e_{33}=15\times10^{-9}$ F/m	$l_{\rm act} = 10$ mm	$t_{\rm act} = 0.63$ mm
		Composite 1	MFC-P1 patch		
$Y_1^E = 30$ GPa	$Y_2^E = 7.65$ GPa	$\nu_{12} = 0.31$		$d_{11} = 404 \times 10^{-12}$ m/V	$d_{12} = -168 \times 10^{-12}$ m/V
$e_{11} = 15 \times 10^{-9}$ F/m	β=0.5	<i>a</i> =0.1 mm		$l_{\rm act}$ =10 mm	$t_{\rm act} = 0.2 \text{ mm}$

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#### B. Sensor output

This subsection will derive the voltage output generated by the sensor as a result of the strain in Eq. (22). The direct piezoelectric effect can be written as (IEEE 1978)

$$\boldsymbol{D} = \boldsymbol{e}\boldsymbol{E} + \boldsymbol{d}\boldsymbol{\sigma},\tag{23}$$

where **D** is the electric displacement vector  $(3 \times 1)$ , **e** is the dielectric permittivity matrix  $(3 \times 3)$ , and **E**, **d**, and **o** represent the electric field, piezoelectric coefficient, and stress components.

#### 1. Monolithic PZT sensor

The voltage response of a surface-mounted PZT sensor subjected to ultrasonic waves was derived in a recent work (Lanza di Scalea *et al.* 2007a). This work considered a monolithic PZT patch  $(d_{31}=d_{32})$  poled along the thickness direction 3  $(E_1=E_2=0)$ , under plane stress conditions (thin sensor,  $\sigma_{33}=\sigma_{13}=\sigma_{23}=0$ ), and modeled as an open circuit  $(\int D_3 dA=0)$ . This previous work neglected shear-lag effects; hence, the strain in the sensor  $\varepsilon_{sens}$  was considered to be equal to the Lamb strain in the substrate plate  $\varepsilon_x$  of Eq. (17). The work is here extended to account for shear lag, therefore, using Eq. (22) for the strain transferred to the sensor. The voltage output  $V_{PZT}$  is found by integrating the in-plane strain  $\varepsilon_{sens}$  along the sensor length,

$$V_{\rm PZT} = S_{\rm PZT} \int_{-l_{\rm PZT}/2}^{l_{\rm PZT}/2} \varepsilon_{\rm sens} dx, \qquad (24)$$

where the sensitivity term (Lanza di Scalea et al. 2007a) is

$$S_{\rm PZT} = \frac{t_{\rm PZT}}{l_{\rm PZT}} \frac{d_{31}Y^E}{[e_{33}(1-\nu) - 2d_{31}^2Y^E]}.$$
 (25)

In this expression,  $Y^E$  and  $\nu$  are Young's modulus and Poisson's ratio, respectively, of the PZT material;  $l_{PZT}$  and  $t_{PZT}$  are the sensor length and thickness.

# 2. Composite MFC sensor

Referring to Fig. 2(b), the MFC poling is along the fiber direction 1 ( $E_2=E_3=0$ , the nonzero piezoelectric coefficients are  $d_{11}$  and  $d_{12}$ , and the nonzero dielectric permittivity coefficient is  $e_{11}$ . The nonzero charge density term  $D_1$  can, thus, be written as

$$D_{1} = \begin{bmatrix} a_{11} & a_{12} & 0 \end{bmatrix} \\ \times \begin{bmatrix} Y_{1}^{E} / (1 - \nu_{12}\nu_{21}) & \nu_{12}Y_{2}^{E} / (1 - \nu_{12}\nu_{21}) & 0 \\ \nu_{12}Y_{2}^{E} / (1 - \nu_{12}\nu_{21}) & Y_{2}^{E} / (1 - \nu_{12}\nu_{21}) & 0 \\ 0 & 0 & G_{12} \end{bmatrix} \\ \times \begin{bmatrix} \varepsilon_{\text{sens}} - E_{1}d_{11} \\ -E_{1}d_{12} \\ 0 \end{bmatrix} + e_{11}E_{1}, \qquad (26)$$

where  $\varepsilon_{\text{sens}}$  is the in-plane strain in the MFC patch assumed along the fiber direction 1, and  $Y_i^E$ ,  $\nu_{ij}$ , and  $G_{12}$  are the inplane orthotropic elastic constants of the MFC patch.

To derive the voltage response, the equivalent model shown in Fig. 2(c) is considered. Here, the MFC is reduced to an equivalent PZT with thickness equal to the MFC electrode spacing *a* and length equal to the MFC net electrode length ( $pl_e$ ), where *p* is the number of electrodes on a single face of the MFC. The net electrode length is, thus, a fraction of the full MFC length,  $pl_e = \beta l_{MFC}$ , where  $\beta$  is a reduction factor depending on the design of the MFC. The equivalent model also requires a rotation of the sensor axes (1,3), as shown in Fig. 2(c).

Considering the equivalent model as an open circuit,

$$\int \int D_1 dx dy = 0, \tag{27}$$

and the resulting generated voltage  $V_{\rm MFC}$  is given by the integral of the electric field divided by the sensor area,

$$V_{\rm MFC} = -\frac{\int \int \int E_1 dx dy dz}{\beta l_{\rm MFC} b_{\rm MFC}},$$
(28)

where  $b_{\rm MFC}$  is the width of the patch. Solving Eq. (26) for  $E_1$ , and substituting into Eq. (28), yields the voltage generated by the P1-type MFC sensor  $V_{\rm MFC}$  as a function of the in-plane strain in the sensor  $\varepsilon_{\rm sensy}$ 

$$V_{\rm MFC} = S_{\rm MFC} \int_{-l_{\rm MFC}/2}^{l_{\rm MFC}/2} \varepsilon_{\rm sens} dx, \qquad (29)$$

where the sensitivity term is

$$S_{\rm MFC} = \frac{a}{\beta l_{\rm MFC}} \frac{(d_{11}Y_1^E + d_{12}\nu_{12}Y_2^E)}{[e_{11}(1 - \nu_{12}\nu_{21}) - (d_{11}^2Y_1^E + 2d_{11}d_{12}\nu_{12}Y_2^E + d_{12}^2Y_2^E)]}$$

(30)

This result is similar to that derived previously for a P2-type MFC sensor (Matt and Lanza di Scalea 2007) with the substitutions of the P1-sensor equivalent thickness  $t_{MFC} \rightarrow a$  and P1-sensor equivalent length  $l_{MFC} \rightarrow \beta l_{MFC}$ .

#### VI. COMPLETE ACTUATOR-SENSOR RESPONSE

The complete pitch-catch solution for a pair of PZT or MFC transducer patches (Fig. 1) can now be found by substituting the strain expression of Eq. (22) into the sensor response of Eqs. (24) and (29). Indicating by  $V_{PZT \rightarrow PZT}$  the

voltage generated by a PZT actuator-sensor pair and by  $V_{\text{MFC}\rightarrow\text{MFC}}$  the voltage generated by a MFC actuator-sensor pair, the following result is obtained:

$$V_{\text{PZT}\to\text{PZT,MFC}\to\text{MFC}} = S_{\text{PZT,MFC}} \int_{-l_{\text{sens}}/2}^{l_{\text{sens}}/2} \varepsilon_{\text{sens}} dx$$
  
$$= i S_{\text{PZT,MFC}} \varepsilon_x \left(\frac{\Psi}{\alpha + \Psi}\right) \int_{-l_{\text{sens}}/2}^{l_{\text{sens}}/2} \left[1 - \frac{\cosh(2\Gamma x/l_{\text{sens}})}{\cosh\Gamma}\right]$$
  
$$\times e^{i(kx - \omega t)} dx, \qquad (31)$$

where the sensor sensitivities,  $S_{PZT}$  or  $S_{MFC}$ , are given in Eqs. (25) or (30), and the actuated Lamb wave amplitude  $\varepsilon_x$  is given in Eq. (17). The shear-lag parameter  $\Gamma$  and the plate/sensor stiffness ratio  $\Psi$  are given in Eqs. (6) and (7) with the substitution of the actuator properties ( $Y_{act}^E$ ,  $l_{act}$ ,  $t_{act}$ ) with the corresponding sensor properties ( $Y_{sens}^E$ ,  $l_{sens}$ ,  $t_{sens}$ ). The integral in Eq. (31) can be solved in closed form, similar to what was done in Eq. (11) for the Fourier transform of the actuation shear stress. This requires making a variable substitution  $x \rightarrow 2\Gamma x/l_{sens}$ , which yields the following final voltage expression:

$$V_{\text{PZT}\to\text{PZT,MFC}\to\text{MFC}} = iS_{\text{PZT,MFC}} \varepsilon_x \left(\frac{\Psi}{\alpha + \Psi}\right) (R_{\text{perf-bond}} - R_{\text{shear-lag}}) e^{-i\omega t}, \quad (32)$$

where it was possible to isolate the two terms responsible for the perfect-bond response  $R_{perf-bond}$  and for the shear-lag response  $R_{shear-lag}$  given by

$$R_{\text{perf-bond}} = \frac{2\sin(kl_{\text{sens}}/2)}{k},$$
(33)

$$R_{\text{shear-lag}} = \frac{4l_{\text{sens}}\Gamma}{(k^2 l_{\text{sens}}^2 + 4\Gamma^2)} [\tanh(\Gamma)\cos(kl_{\text{sens}}/2) + (kl_{\text{sens}}/2\Gamma)\sin(kl_{\text{sens}}/2)].$$
(34)

From Eq. (32), the general pitch-catch response emerges as a combination of the perfect-bond response and the shear-lag response whose effect is to decrease the overall signal amplitude (notice the minus sign for  $R_{\text{shear-lag}}$ ).

The case of perfect strain transfer between the plate and the sensor is retrieved by letting  $\Gamma \rightarrow \infty$  (the shear-lag term disappears as  $\lim_{\Gamma \rightarrow \infty} R_{\text{shear-lag}}=0$ ) and by assuming an ideal sensor [thin and compliant sensor such that  $E_{\text{plate}}t_{\text{plate}}$  $>>E_{\text{sens}}t_{\text{sens}}$  or  $\Psi>>1$ , hence,  $\Psi/(\alpha+\Psi)\simeq 1$ ]. Under these perfect-sensor conditions, Eq. (32) simplifies to

$$V_{\text{perf-bond}} = iS\varepsilon_x \frac{2\sin(kl_{\text{sens}}/2)}{k} e^{-i\omega t},$$
(35)

which is equivalent to the result obtained previously for piezoelectric wave sensors under perfect strain-transfer assumptions (Lanza di Scalea *et al.* 2007a; Matt and Lanza di Scalea 2007).

The pitch-catch response magnitude from Eq. (32) is plotted in Fig. 7(a) for a PZT pair and in Fig. 7(b) for a MFC pair as a function of the nondimensional wave number



FIG. 7. Pitch-catch response to the fundamental  $S_0$  and  $A_0$  modes in a 1.58-mm-thick aluminum plate and (a) monolithic PZT actuator-sensor pair and (b) composite MFC-P1 actuator-sensor pair.

(kl/2). The lengths of actuators and sensors are assumed to be equal  $(l=l_{act}=l_{sens}=10 \text{ mm})$ . The thicknesses are  $t_{act}$  $=t_{sens}=0.63$  mm for the PZTs and  $t_{act}=t_{sens}=0.2$  mm for the MFCs. The excitation voltage is 1 V. All other properties are listed in Table I. Notice that the PZT pair is more sensitive to  $S_0$  than to  $A_0$ , and the opposite is true for the MFC pair. This result is similar to what seen in Fig. 6 which only considered actuation. Wavelength tuning conditions are still evident at points of zero and maximum response. However, the pitchcatch model predicts a drop in the response with increasing kl values, as a result of the decreasing  $S_0$  and  $A_0$  strain amplitudes at the surface of the plate with increasing frequency. Finally, it is interesting to note that the response of the PZT pair is similar in magnitude to that of the MFC pair. Thus, the effect of the one order of magnitude stronger MFC actuation seen in the previous Fig. 6 is compensated by the smaller MFC sensing ability. This is partly due to the disappearance of the MFC interelectrode spacing a and PZT thickness t from the pitch-catch response in Eq. (32) (these terms are at the denominator of the actuation strains [Eqs. (3) and (5)] and at the numerator of the sensitivity terms [Eqs. (25) and (30)]).

# **VII. TEMPERATURE EFFECTS**

The expression in Eq. (32) allows isolating the temperature dependence of the properties of actuator, sensor, bond layers, and plate substrate. These properties are examined in the following subsections considering the typical aircraft range of -40-+60 °C. Unless otherwise specified, a linear dependence of each property on temperature is assumed,

$$P(T) = P(T_0) + \frac{\partial P(T)}{\partial T} \Delta T, \qquad (36)$$

where *P* represents one of the properties, *T* is the generic temperature,  $T_0$  is the ambient temperature 20 °C, and  $\partial P / \partial T$  is the sensitivity to temperature.

# A. Temperature effects on actuator and sensor properties

The temperature-dependent properties of the PZT or MFC actuator and sensor patches influencing the response in Eq. (32) are the following: Young's moduli  $Y^E(PZT)$ ,  $Y_1^E$ ,  $Y_2^E(MFC)$  from Eqs. (6), (7), (25), and (30); Poisson's ratios  $\nu(PZT)$ ,  $\nu_{12}$ ,  $\nu_{21}(MFC)$  from Eqs. (25) and (30); piezoelectric coefficients  $d_{31}(PZT)$ ,  $d_{11}$ ,  $d_{12}(MFC)$  from Eqs. (3), (5), (25), and (30); dielectric permittivity terms  $e_{33}(PZT)$ ,  $e_{11}(MFC)$  from Eqs. (25) and (30); and length and thickness dimensions. All of these terms are considered in the following.

#### 1. Monolithic PZT patch

Type 5A PZT material is considered. Young's modulus and Poisson's ratios were modeled according to Eq. (36) by using  $Y^E(T_0)=70$  GPa,  $\nu(T_0)=0.31$ , and temperature sensitivities of  $\partial Y^E/\partial T=0.16$  GPa/°C and  $\partial \nu/\partial T=-0.013/$ °C (Sherrit *et al.* 1999).

The piezoelectric coefficient was also modeled linearly assuming  $d_{31}(T_0) = -168 \times 10^{-12} \text{ m/V}$  and sensitivity  $\partial d_{31}/\partial T = -0.5 \text{ m/V} \,^{\circ}\text{C}$  (Lee and Saravanos 1998). The dielectric permittivity term was considered to be bilinear to best fit the data by Lee and Saravanos (1998) will be also discussed later and shown in Fig. 11. The ambient value was  $e_{33}(T_0) = 15 \times 10^{-9} \text{ F/m}$ , with sensitivities  $\partial e_{33}/\partial T = 0.043 \times 10^{-9} \text{ F/m} \,^{\circ}\text{C}$  below ambient temperature and  $\partial e_{33}/\partial T$  $= 0.14 \times 10^{-9} \text{ F/m} \,^{\circ}\text{C}$  above ambient temperature. Patch length and thickness changes were modeled assuming a thermal expansion coefficient for PZT 5A of  $3 \times 10^{-6} \text{ m/m} \,^{\circ}\text{C}$ (Lee and Saravanos 1998).

#### 2. Composite MFC patch

Conventional rules of mixture were used to calculate the effective Young's moduli and Poisson's ratios of the MFC patches from the constituent properties of fibers  $(E_f, v_f)$  and matrix  $(E_m, v_m)$  through the fiber volume fraction  $V_f$  (Jones 1975),

$$Y_{1}^{E} = E_{f}V_{f} + E_{m}(1 - V_{f}), \quad Y_{2}^{E} = \frac{E_{f}E_{m}}{(1 - V_{f})E_{f} + V_{f}E_{m}},$$

$$v_{12} = (1 - V_{f})v_{m} + V_{f}v_{f}, \quad v_{21} = v_{12}\frac{Y_{2}^{E}}{Y_{1}^{E}}.$$
(37)

The temperature dependence of  $E_m$  and  $v_m$  for the PZT-5A fibers was modeled as in the previous Sec. VII A 1. The matrix properties were obtained experimentally from longitudinal and shear ultrasonic time-of-flight measurements conducted on epoxy coupons in an environmental chamber with temperature control between -40 and +60 °C. The measure-

ments were averaged over two separate coupons and three temperature cycles. The following matrix properties were obtained:  $E_m(T_0)=4.7$  GPa and  $v_m(T_0)=0.45$ , with sensitivities of  $\partial E_m/\partial T=-0.028$  GPa/°C and  $\partial v_m/\partial T=0.009/°C$ . A constant volume fraction  $V_f=0.4$  was assumed. The temperature sensitivity of the MFC properties  $Y_1^E$ ,  $Y_2^E$ ,  $v_{12}$ , and  $v_{21}$  was then calculated from Eq. (37).

Regarding the piezoelectric coefficients, the following values were used:  $d_{11}(T_0) = 404 \times 10^{-12} \text{ m/V}$ ,  $d_{12}(T_0) = -167 \times 10^{-12} \text{ m/V}$ , as obtained from manufacturer's data; sensitivities  $\partial_{d_{11}}/\partial T = 1 \text{ m/V} \circ \text{C}$  from Hooker (1998);  $\partial d_{12}/\partial T$  was assumed equal to  $\partial d_{31}/\partial T$  of the previous Sec. VII A 1 (as  $d_{12}$  of the MFC fibers is equivalent to  $d_{31}$  of the monolithic PZT). The dielectric permittivity trend for  $e_{11}(T)$  was also set equal to the trend  $e_{33}(T)$  of the previous section, with  $e_{11}(T_0) = 15 \times 10^{-9} \text{ F/m}$  ( $e_{11}$  of the MFC fibers is equivalent to  $e_{33}$  of the monolithic PZT). Patch length and thickness changes were modeled assuming a coefficient of thermal expansion of  $5 \times 10^{-6} \text{ m/m} \circ \text{C}$  along the fibers and 15  $\times 10^{-6} \text{ m/m} \circ \text{C}$  across the fibers (Williams *et al.* 2004).

# B. Temperature effects on transducer/plate bond layer properties

The bond layer enters the final Eq. (32) through its shear modulus  $G_{\text{bond}}$  and thickness  $t_{\text{bond}}$ , both present in Eqs. (6) and (18). Ambient-cure epoxy adhesive (Loctite from Henkel Corporation, Rocky Hill, CT) was used in this study. The temperature dependence of  $G_{\text{bond}}$  was measured by ultrasonic shear-wave velocity measurements on epoxy coupons tested in the autoclave between -40 and +60 °C. By averaging the measurements over two separate coupons and three temperature cycles, the following regression line was obtained with a 0.96 regression coefficient:

$$G_{\text{bond}}(T) \ (GPa) = 3.2 - 0.013T \ (^{\circ}\text{C})$$
 (38)

Changes in thickness of the bond due to thermal expansion were considered negligible and ignored.

#### C. Temperature effects on substrate plate properties

Properties of the substrate plate affecting the pitch-catch response include Young's modulus  $E_{\text{plate}}$ , thickness  $t_{\text{plate}}$  from Eq. (7), and shear modulus  $G_{\text{plate}}$  from Eq. (17). In addition,  $G_{\text{plate}}$  and density  $\rho_{\text{plate}}$  will affect the bulk longitudinal wave velocities, hence, the Lamb dispersion relations  $k(\omega)$ . The following values were considered for aluminum 2024:  $E_{\text{plate}}(T_0) = 70 \text{ GPa}, \quad G_{\text{plate}}(T_0) = 27 \text{ GPa}, \quad \rho_{\text{plate}}(T_0)$ temperature  $=2700 \text{ kg/m}^3$ , and sensitivities of  $\partial E_{\text{plate}} / \partial T = -0.04 \text{ GPa} / ^{\circ}\text{C} \text{ and } \partial G_{\text{plate}} / \partial T = -0.015 \text{ GPa} / ^{\circ}\text{C}.$ The nominal coefficient of thermal expansion for aluminum of  $2.31 \times 10^{-5}$  m/m °C was assumed for the  $t_{\text{plate}}$  changes.

Figure 8 shows the dispersion curves  $k(\omega)$  for  $S_0$  and  $A_0$ in a 1.58-mm-thick aluminum plate at the different temperatures of -40, +20, and +60 °C. As expected, the temperature increase causes a decrease in phase velocities  $\omega/k$  for both modes. It can also be seen that, for the temperature range of interest, the shifts in  $k(\omega)$  are quite small. Hence, little change should be expected on the wavelength tuning points of the pitch-catch response spectrum.



FIG. 8. Dispersion curves for the fundamental Lamb modes in a 1.58-mm-thick aluminum plate at the temperatures of -40, +20, and +60 °C.

### **D. Results**

Results from the pitch-catch model of Eq. (32), with the temperature dependences discussed in Secs. VII A–VII C, are shown in Fig. 9 for the  $S_0$  mode and in Fig. 10 for the  $A_0$  mode. The amplitude of the PZT-PZT response  $V_{\text{PZT}\rightarrow\text{PZT}}$  and the MFC-MFC response  $V_{\text{MFC}\rightarrow\text{MFC}}$  is plotted against the nondimensional wave number (kl/2) as a function of temperature (below ambient,  $-40 \degree \text{C} \le T \le +20 \degree \text{C}$ , or above ambient,  $+20 \degree \text{C} \le T \le +60 \degree \text{C}$ , every  $10 \degree \text{C}$ ). It is assumed that  $l=l_{\text{act}}=l_{\text{sens}}$  and a unity voltage is applied to the actuator,

 $V_{appl}=1$  V. The plots show that the main effect of temperature is a change in the response amplitude, caused by both actuator/plate and plate/sensor systems. The shifts in wavelength tuning points (zeroes and maxima of the response spectra), due to shear lag and to the  $k(\omega)$  shifts of Fig. 8, are negligible. The changes in response amplitude between the highest (+60 °C) or the lowest (-40 °C) temperature and ambient temperature are on the order of 20% in the first response peak (at  $kl/2 \approx 1.2$ ). With increasing kl values, the temperature-induced amplitude variations decrease as a result of the drop in the overall response caused by the decreasing  $S_0$  and  $A_0$  strain amplitudes with increasing frequency.

Most interestingly, the response amplitude follows two opposite trends at opposite sides of the ambient temperature: decreasing response with increasing temperature in +20 °C  $\leq T \leq +60$  °C and increasing response with increasing temperature in -40 °C  $\leq T \leq +20$  °C. These trends are consistent for both PZT and MFC pairs and for  $S_0$  and  $A_0$  modes. The reason for the opposite trends should be attributed to the competing roles of the piezoelectric coefficients and the dielectric permittivity terms. By isolating the effect of these two parameters and considering the PZT-PZT case, the pitchcatch response amplitude of Eq. (32) is proportional to the following expression:



FIG. 9. Pitch-catch response to  $S_0$  Lamb mode in a 1.58-mm-thick aluminum plate for [(a) and (c)] PZT actuator-sensor pair and [(b) and (d)] MFC actuator-sensor pair, under changing temperature in the ranges [(a) and (b)] above ambient (+20 + 60 °C), and [(c) and (d)] below ambient (-40 - +20 °C).



FIG. 10. Pitch-catch response to  $A_0$  Lamb mode in a 1.58-mm-thick aluminum plate for [(a) and (c)] PZT actuator-sensor pair and [(b) and (d)] MFC actuator-sensor pair, under changing temperature in the ranges [(a) and (b)] above ambient (+20 + 60 °C) and [(c) and (d)] below ambient (-40 + 20 °C).

$$|V_{\text{PZT}\to\text{PZT}}| \propto \frac{d_{31}^2}{e_{33}(1-\nu) - 2d_{31}^2 Y_E},$$
 (39)

where  $d_{31}^2$  at the numerator results from the product of  $d_{31}$  in the actuator's strain expression of Eq. (3) and  $d_{31}$  in the sensor's strain expression of Eq. (25). From Eq. (39), the competing roles are now clear, since a larger  $d_{31}$  generates a stronger  $|V_{\text{PZT}\rightarrow\text{PZT}}|$  response, whereas a larger  $e_{33}$  generates a weaker  $|V_{\text{PZT}\rightarrow\text{PZT}}|$  response. The trends of  $d_{31}$  and  $e_{33}$  as a function of temperature for PZT-5A are shown in Fig. 11



FIG. 11. Piezoelectric coefficient  $d_{31}$  and dielectric permittivity  $e_{33}$  relative to ambient temperature (+20 °C) values, as a function of temperature for PZT-5A.

(Lee and Saravanos 1998). Both parameters, in absolute values, increase with increasing temperature. However, while the rates of change for  $d_{31}$  and  $e_{33}$  are comparable below ~20 °C, the rate of change for  $e_{33}$  substantially increases above ~20 °C. Consistently with the results of Figs. 9 and 10, for the given PZT material, the role of  $d_{31}$  on  $|V_{PZT \rightarrow PZT}|$  is dominant below 20 °C (i.e., stronger response with increasing temperature); the role of  $e_{33}$  becomes, instead, dominant above 20 °C (i.e., weaker response with increasing temperature). Similar arguments hold for the MFC-MFC pair, which uses the same PZT-5A fibers.

#### E. Experiment

The theory was compared to pitch-catch measurements taken on 1.58-mm-thick aluminum plates [Fig. 12(a)]. The



FIG. 12. Experimental tests: (a) aluminum plate with MFC actuator-sensor pair and (b) environmental chamber with temperature control.



FIG. 13. Pitch-catch response to  $S_0$  Lamb mode in a 1.58-mm-thick aluminum plate for the PZT actuator-sensor pair under changing temperature in the ranges [(a) and (b)] above ambient (+20-+60 °C) and [(c) and (d)] below ambient (-40-+20 °C). Comparison between [(a) and (c)] model and [(b) and (d)] experiment.

PZT patches used for the tests (American Piezo Corporation, Mackeyville, PA) were type 5A disks with a diameter of 10 mm and other properties listed in Table I. The MFC patches were type P1 rectangular transducers (Smart Materials Corporation, Saratoga, FL) with an active length of 28 mm and other properties also listed in Table I. The transducers were bonded to the plates by using the ambient-cure epoxy adhesive from Loctite, which was characterized in Sec. VII B. The actuator-sensor distance was kept at 100 mm for both PZT pair and MFC pair. The measurements were taken in an environmental chamber [Fig. 12(b)] between -40and +60 °C at 10 °C intervals. Thermocouples were attached to the plates to verify that thermal equilibrium was achieved at each temperature step. A 3.5-cycle, Hanningmodulated toneburst was used as the actuating signal. The excitation was swept between 100 and 500 kHz. The dominant  $S_0$  mode was received and analyzed. The root-mean square of the detected  $S_0$  was computed at each excitation frequency to quantify signal strength.

The results are plotted in Fig. 13 for the PZT pair and in Fig. 14 for the MFC pair. The corresponding results from the model in Eq. (32) are also shown for comparison. The data are plotted separately for the above ambient temperatures  $(+20 \ ^\circ C \le T \le +60 \ ^\circ C)$  and the below ambient temperatures

 $(-40 \circ C \leq T \leq +20 \circ C)$ . The plots are normalized to the relative maxima; since the model neglected wave damping in the plate, measured and predicted absolute amplitudes cannot be compared.

The plots show the expected trends seen in the previous figures of decreasing response with increasing temperature above 20 °C and increasing response with increasing temperature below 20 °C. As discussed in the previous section, the competing roles of piezoelectric coefficients and dielectric permittivity terms are responsible for the opposite trends in the two ranges. The agreement between model and experiment is quite satisfactory for both the PZT-PZT pair (Fig. 13) and the MFC-MFC pair (Fig. 14). The wavelength tuning points match to within a few percent. The model for the PZT pair overestimates the response amplitude at the lower frequencies of 100-200 kHz. At the points of peak responses, the discrepancy between model and experiment is within 5% for the PZT pair and within 10% for the MFC pair. Overall, the results demonstrate that the proposed model captures well the general trend of a pitch-catch measurement under changing temperature within the -40-+60 °C range of interest.



FIG. 14. Pitch-catch response to  $S_0$  Lamb mode in a 1.58-mm-thick aluminum plate for the MFC actuator-sensor pair under changing temperature in the ranges [(a) and (b)] above ambient (+20-+60 °C), and [(c) and (d)] below ambient (-40-+20 °C). Comparison between [(a) and (c)] model and [(b) and (d)] experiment.

# **VIII. CONCLUSIONS**

A model for predicting the response of Lamb wave pitch-catch SHM systems has been developed. The model combines the piezomechanical properties of the actuator and the sensor patches, the interaction between the actuator and sensor with the substrate plate, and the Lamb wave dispersion in the plate. The formulation explicitly accounts for the relevant temperature-dependent parameters of the system, including the transducer piezoelectric and dielectric permittivity properties, the transducer-to-plate adhesive bond properties in a shear-lag framework, and the dispersion curves of the plate. One of the key steps has been deriving a closedform solution for the spatial Fourier transform of the shear stress transferred from the actuator into the plate under shear lag. The Fourier transform expression enables the use of mode expansion to predict the Lamb waves generated in the plate. The output voltage from the sensor has been derived by accounting for the actual strain transferred from the plate, again under shear lag. The cases of monolithic PZT patches and composite MFC patches on aluminum plates have been

considered. The focus of the study has been the temperature range of -40-+60 °C, which is typical of aircraft operations.

On the wave actuator side, the shear-lag results indicate that changing bond layer stiffness has a small effect on the wavelength tuning points and a similarly small effect on the net shear force transferred to the plate. Changing actuator stiffness, instead, has a marked effect on the net shear force transferred to the plate, with stiffer actuators producing larger forces. Comparing PZT-5A and MFC type P1 actuator patches of the same length and under the same excitation voltage, the latter produces  $S_0$  and  $A_0$  Lamb wave strain amplitudes, which are one order of magnitude larger than the former. This is due to the small interelectrode spacing of the MFC, coupled with the MFC's larger piezoelectric constant  $d_{11}$  compared to the PZT's  $d_{31}$ . However, once the transducers are used in a pitch-catch actuator-sensor pair, the  $S_0$  and  $A_0$  responses of the PZT pair become comparable in magnitude to those of the MFC pair. This is partly due to the disappearance of the MFC interelectrode spacing term.

By letting all relevant parameters in the model change with temperature, the pitch-catch  $S_0$  and  $A_0$  response spectra for PZT and MFC pairs on an aluminum plate were calculated between -40 and +60 °C. It was found that the changing temperature has a negligible effect on the wavelength tuning points and a marked effect on the response amplitude. The changes in response amplitude, relative to the ambient temperature value, were predicted as high as 20% at the extreme temperatures of -40 and +60 °C. These changes affect mainly the primary response peak (first wavelength tuning point near  $kl = \pi$ ) and become less relevant as kl (i.e., frequency) increases. Experimental measurements followed the same behavior. Limited discrepancies were found between theory and experiment, more markedly so for the MFC case. It was also consistently observed that the trend in response amplitude switches at the ambient temperature, with stronger responses with increasing temperature for  $-40 \,^{\circ}C \leq T \leq$ +20 °C and weaker responses with increasing temperature for +20 °C  $\leq T \leq$  +60 °C. This is the result of the competing roles of the piezoelectric coefficients and the dielectric permittivity terms (larger d result in stronger response; larger e result in weaker response). For the PZT-5A material examined, the temperature sensitivity of the dielectric permittivity increases above ambient temperature, causing the weakening of the overall response.

This study can be used to develop strategies for compensation of the temperature effects in guided-wave damage detection systems. The results shown apply to PZT-5A and MFC type P1 transducers bonded on aluminum plates with ambient-cure epoxy. However, the model can be easily extended to other systems by inputting the appropriate piezoelectric, dielectric, and mechanical properties of transducers, adhesives, and plates. The limiting requirements of the formulation are those of thin transducer patches (plane stress in the plate's surface), undamped isotropic plates (Lamb modes), straight-crested wave fronts (plane strain in the plate's cross section), and low frequency times plate thickness values (shear-lag theory). The extension of this study to include damping in the plate and circularly crested waves is an ongoing work.

#### ACKNOWLEDGMENTS

Dr. Ivan Bartoli at UC San Diego is acknowledged for obtaining the results shown in Fig. 8. Dr. Howard Matt, formerly at UC San Diego and currently at ATA Engineering in San Diego, CA is acknowledged for the helpful discussions on Lamb wave actuator and sensor models.

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