TEMPERATURE –DEPENDENT COEFFICIENTS OF THERMAL EXPANSION FOR MACRO FIBER COMPOSITE ACTUATORS

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The focus of this research effort is to model the coefficients of thermal expansion for the Macro Fiber Composite actuator as a function of temperature between 0° and 250°C. The required temperature-dependent thermoelastic properties of each constituent material were obtained, and, for the orthotropic layers, the coefficients of thermal expansion were calculated using a variety of micromechanics models. The alternative rule of mixtures formula was selected as the most accurate based on a comparison with ANSYS finite element models. With the temperature-dependent properties known for each layer, equations for the two coefficients of thermal expansion of the entire actuator were derived using Classical Lamination Theory. These results were seen to agree closely with an ANSYS finite element model of the unit cell of the Macro Fiber Composite actuator.

Keywords: *Macro Fiber Composites, Temperature-Dependent Properties, Piezoceramic Fibers, CTE*

1 Introduction

Macro Fiber Composite (MFC) actuators are part of an emerging technology that strives to improve the state of the art for structural actuation, which currently relies heavily upon monolithic piezoceramic materials. The MFC is composed of unidirectional piezoceramic fibers of rectangular cross-section that are embedded in a thermosetting polymer matrix and sandwiched between Kapton© sheets layered with copper interdigitated electrodes. This electrode pattern, along with the hybrid-composite lamination scheme allows for a robust yet flexible actuator that utilizes the "d_{33}" piezoelectric effect, thus inducing much higher strain levels than traditional piezoceramics.

The MFC actuator has tremendous potential for improving the performance of aerospace structures such as helicopter rotor blades, fighter jet tailfins, morphing wings and telecommunication satellites. Each of these applications could subject the actuators to extremely wide temperature ranges where thermoelastic behavior could become significant. Although the intended purpose of the MFC is actuation through applied voltage, this research effort deals only with the constant-field (short circuit) thermoelastic properties while developing equations for the coefficients of thermal expansion (CTE) of the MFC as a function of temperature. Pyroelectric effects are also assumed small and therefore neglected. The temperature range of interest is from 0° to 250°C, and the analytical results are verified using the ANSYS finite element software.

2 Constituent Material Properties

The first step in the current analysis was to obtain all of the required thermoelastic properties as a function of temperature for each of the five constituent materials, namely Kapton, acrylic, copper, epoxy (all isotropic) and PZT (transversely isotropic). These values were obtained from manufacturers [1, 2], when available, or from materials handbooks [3] or other research institutions [4, 5, 6]. Each property (elastic moduli and CTE) was represented as an algebraic, temperature-dependent equation (Poisson's ratio was assumed constant with temperature). In the interest of brevity, these equations are omitted. However, the elastic modulus for the four isotropic materials was found to decrease linearly with temperature, by 30, 50, 50 and 70%, respectively, over the temperature range of interest. The CTEs of the polymeric materials were constant at lower temperatures, and then underwent a step increase at the glass transition temperature of the particular material. However, the CTE of copper increased linearly by 13% over the temperature range. As for the transversely isotropic PZT fibers, the elastic

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moduli are not highly temperature-dependent, but were represented as decreasing linearly by 5% over the temperature range of interest. In the poling direction of the PZT material (fiber direction), the CTE decreases quadratically, and becomes negative above 175°C, while transverse to the fiber direction, the CTE is always positive and simply increases linearly.

3 Orthotropic Layers

Figure 1 shows the four basic layers that comprise the MFC actuator and the geometric Cartesian coordinate system used in the forthcoming analysis.

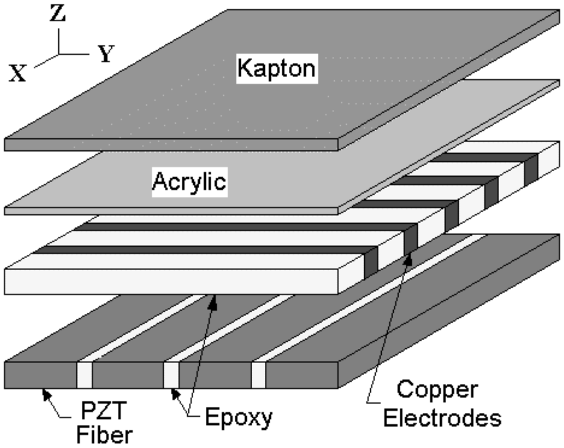

Figure 1: Layers of the MFC actuator (symmetric about the middle of the PZT/epoxy layer).

For the Kapton and acrylic, the isotropic material properties described above and the geometric properties listed in Table 1 are sufficient to describe the behavior of their respective layers.

Table 1: MFC Laminae Geometric Properties

Property	Value
Kapton Thickness, μm	25.40
Acrylic Thickness, μm	12.70
Copper Thickness, μm	17.78
PZT Thickness, μm	127.00
PZT Fiber Width, μm	355.60
PZT Fiber Kerf, μm	76.20
Copper Fiber Width, μm	101.60
Copper Fiber Kerf, μm	431.80
PZT Fiber Volume Fraction	0.824
Copper Fiber Volume Fraction	0.190

However, the thermoelastic properties of the orthotropic copper/epoxy and PZT/epoxy laminae must be determined using a micromechanics approach. Each longitudinal modulus and Poisson's ratio is calculated using the well known rule of mixtures formulas, while the transverse and shear moduli are calculated using the Chamis equation and an elasticity solution, respectively. Previous experiments show these equations to predict these properties reasonably well for the MFC [7, 8].

The rule of mixtures formula gives the CTE of the orthotropic layers in the fiber direction, α_1, as

$$\alpha_1 = \frac{\left(\alpha_1^f E_1^f - \alpha^m E^m\right)V^f + \alpha^m E^m}{\left(E_1^f - E^m\right)V^f + E^m}. \tag{1}$$

Here, E is the elastic modulus, V represents the volume fraction, superscript f and m denote fiber and matrix properties, respectively and the subscript 1 denotes the fiber direction.

The values predicted using this formula are in close agreement with an ANSYS finite element analysis. However, the transverse CTE of an orthotropic layer is more difficult to predict. Therefore, several micromechanics models were applied to the MFC, including the rule of mixtures, alternative rule of mixtures, Chamis, Shapery and modified strip [9, 10]. A large number of formulas were considered as some are more accurate at higher or lower fiber volume fractions, and, for the MFC geometry, the copper/epoxy layer has a low V^f, while the PZT/epoxy layer has a high V^f. The results of each equation were compared to an ANSYS finite element model of each orthotropic layer, and the alternative rule of mixtures formula was adopted as it provided the most accurate values. Therefore, for the remainder of the current analysis, the CTE in the transverse direction of an orthotropic lamina, α_2, will be taken as

$$\begin{aligned}\alpha_2 &= \alpha^m + \left(\alpha_2^f - \alpha^m\right)V^f \\ &+ \left(\frac{E_1^f \nu^m - E^m \nu_1^f}{E_1}\right)\left(\alpha^m - \alpha_1^f\right)\left(1 - V^f\right)V^f.\end{aligned} \tag{2}$$

Here, the notation is the same as before, with υ denoting Poisson's ratio.

4 Classical Lamination Theory

With all stiffness and thermal expansion properties expressed as a function of temperature, Classical Lamination Theory is applied to the MFC. From a modeling standpoint, the MFC behaves as a symmetric, cross-ply laminate subjected to only a uniform temperature change, ΔT. The free thermal strains of such a laminate are expressed as

$$\begin{Bmatrix} \varepsilon_x^{\circ T} \\ \varepsilon_y^{\circ T} \\ \gamma_{xy}^{\circ T} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{12} & A_{22} & 0 \\ 0 & 0 & A_{66} \end{bmatrix}^{-1} \begin{Bmatrix} N_x^T \\ N_y^T \\ N_{xy}^T \end{Bmatrix} \tag{3}$$

where the thermal force resultants are expressed as

$$\begin{Bmatrix} N_x^T \\ N_y^T \end{Bmatrix} = \sum_{k=1}^{N} \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} \\ \overline{Q}_{12} & \overline{Q}_{22} \end{bmatrix}_k \begin{Bmatrix} \alpha_x \\ \alpha_y \end{Bmatrix}_k \left(z_k - z_{k-1}\right) \Delta T \tag{4}$$

and A_{ij} is the in-plane extensional stiffness matrix for the MFC, $\overline{Q}_{ij}$ is the transformed reduced stiffness matrix of the k^{th} of N total layers, z represents thickness coordinates and superscript T denotes free thermal quantities. Also, it should be noted that $N_{xy}{}^T$ does not appear in Eq. 4 because the symmetric cross-ply nature of the MFC eliminates any induced thermal shear strains, i.e. α_{xy} for all layers is zero. Next, for a unit temperature change, the free thermal strains become the coefficients of thermal expansion, by definition, and Eq. (3) and (4) are combined to give the following formulas for the CTEs of the MFC actuator [9]:

$$\alpha_x^{MFC} = \frac{A_{22}\hat{N}_x^T - A_{12}\hat{N}_y^T}{A_{11}A_{22} - A_{12}^2} \tag{5}$$

$$\alpha_y^{MFC} = \frac{A_{11}\hat{N}_y^T - A_{12}\hat{N}_x^T}{A_{11}A_{22} - A_{12}^2} \tag{6}$$

where

$$\hat{N}_x^T = \sum_{k=1}^{N} \overline{Q}_{11_k}\alpha_{x_k} + \overline{Q}_{12_k}\alpha_{y_k}\left(z_k - z_{k-1}\right) \tag{7}$$

$$\hat{N}_y^T = \sum_{k=1}^{N} \overline{Q}_{12_k}\alpha_{x_k} + \overline{Q}_{22_k}\alpha_{y_k}\left(z_k - z_{k-1}\right) \tag{8}$$

Of course, each of the properties required to calculate both $\overline{Q}_{ij}$ and A_{ij} are known as a function of temperature, thus the CTEs of the MFC given by Eqs. (5, 6) are implicitly dependent on temperature. Due to the lengthy nature and numerous temperature-dependent properties involved in this analysis, the full expressions for these CTEs are omitted; however, the resulting curves generated by *Mathematica* are presented in Figure 4.

5 Finite Element Analysis

In order to verify the forgoing analysis, i.e. Eqs. (5, 6), an ANSYS finite element model was constructed using the appropriate temperature-dependent properties. In developing the model, a unit cell of the MFC was isolated, and then the quarter model of this repeating cell was drawn in ANSYS, as shown in Figure 2.

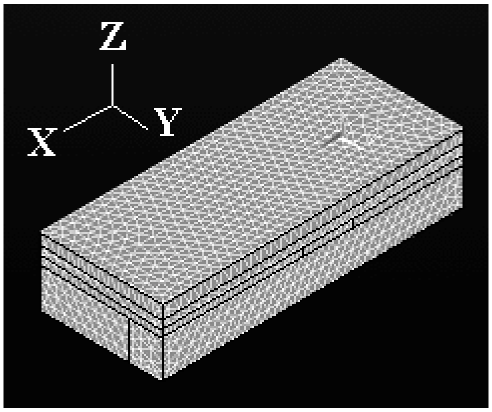

Figure 2: ANSYS Quarter Unit Cell Model of MFC Actuator.

This geometry was meshed with 17,336 ANSYS SOLID185 elements. Since a quarter model was used, symmetric boundary conditions were enforced along the bottom and hidden X-Z surfaces, and the nodal displacements along the top and exposed X-Z faces were coupled in their respective normal directions. A unit thermal load (1°C ΔT) was applied at all nodes, and multiple solutions were run at various temperatures, where the material properties were updated accordingly.

Figure 3 shows the calculated nodal displacement fields in both the x and y directions.

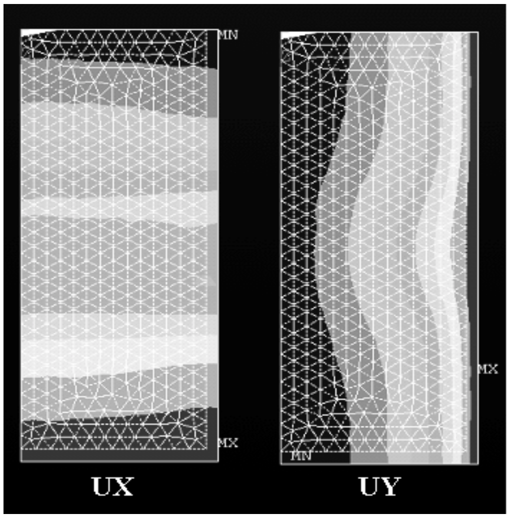

Figure 3: ANSYS x and z Nodal Displacement Fields.

From a list of the maximum nodal displacements, Δx and Δy, the appropriate CTEs were calculated as

$$\alpha_x^{MFC} = \frac{\Delta x}{x * \Delta T} \qquad (9)$$

$$\alpha_y^{MFC} = \frac{\Delta y}{y * \Delta T}. \qquad (10)$$

6 Results and Discussion

Figure 4 depicts the coefficients of thermal expansion for the MFC actuator, α_x^{MFC} and α_y^{MFC}, as a function of temperature from 0° to 250°C. The lines represent the classical lamination theory solution, while the discreet points represent the ANSYS finite element solutions. Clearly, the two methods are in reasonably close agreement.

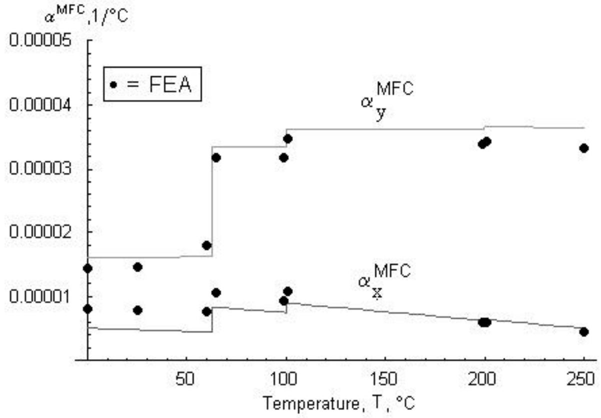

Figure 4: Coefficients of Thermal Expansion for the MFC Actuator.

From this figure, many interesting trends are noted. First, the lower curve, which represents the CTE in the PZT fiber direction, is nearly constant over the entire temperature range and could reasonably be represented as a temperature *independent* property. Since there is a high volume fraction of PZT fibers in the MFC, α_x^{MFC} is dominated by the CTE of the piezoceramic in that direction. This behavior is evident when one notices how α_x^{MFC} decreases between each set of step changes. The steps are evidence of the influence of the epoxy. However, despite the fact that the CTE of the PZT becomes negative at high temperatures, the effects of the other constituent materials maintains a positive α_x^{MFC}. This behavior could be a cause for concern in terms of residual stresses at the interface between the fiber and epoxy matrix. Next, the top curve, which represents the CTE of the MFC transverse to the PZT fibers, does vary significantly over the temperature range of interest. The fact that the large step changes in value closely mimic the behavior of the epoxy is in indication that α_y^{MFC} is a matrix-dominated property.

7 Conclusions

This research effort has presented the coefficients of thermal expansion for the MFC actuator as a function of temperature, based on finite element and classical lamination analyses. It was found that while α_x^{MFC} is nearly constant between 0° and 250°C, α_y^{MFC} does vary significantly, primarily due to the thermoelastic behavior of the epoxy matrix. As a result, the variation of thermoelastic properties with temperature should be carefully modeled, particularly when transverse behavior of the MFC is deemed critical.

8 References

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